

APPLIED ECONOMETRIC TIME SERIES 4TH ED. WALTER ENDERS

Chapter 4

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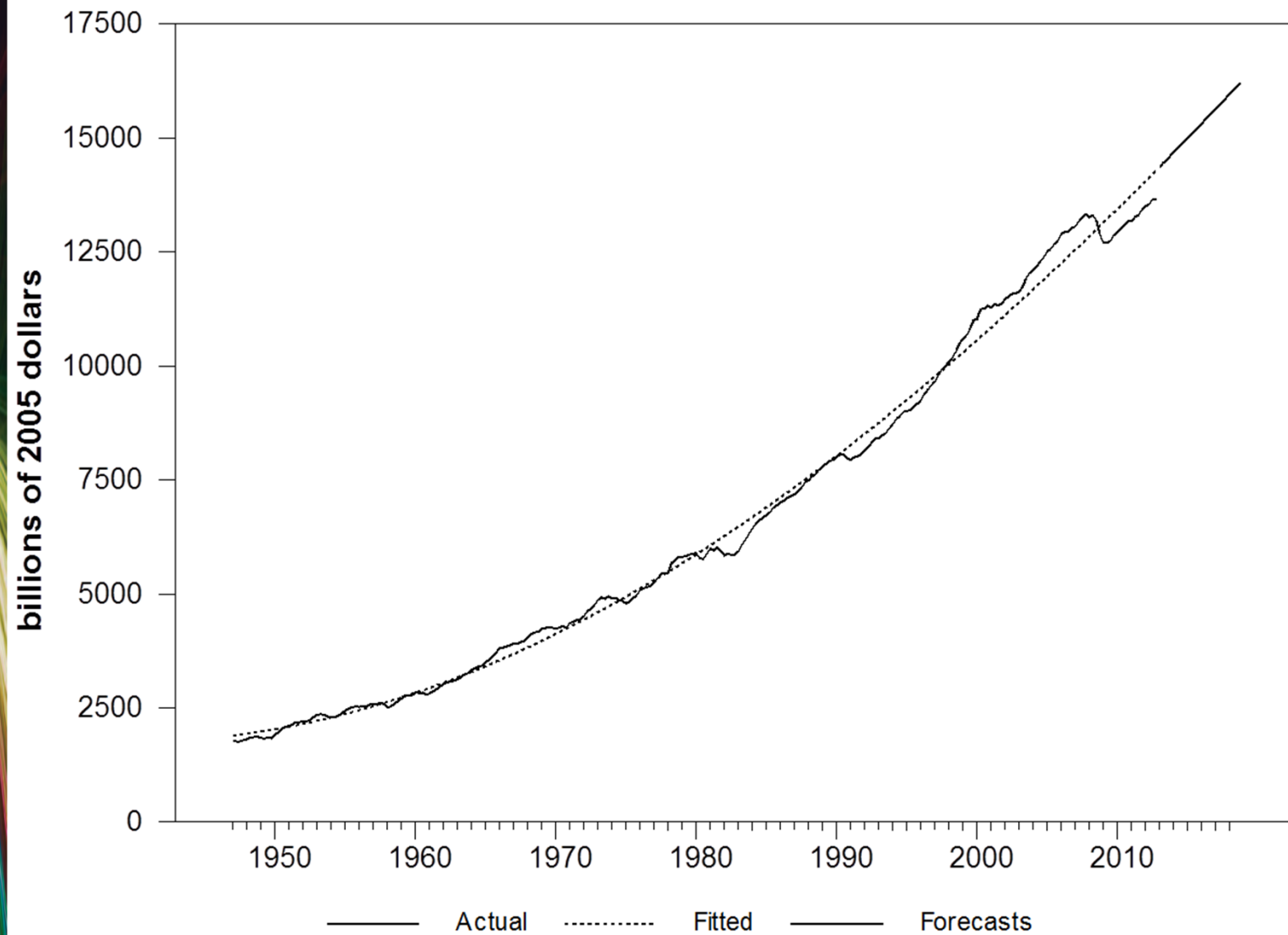


Figure 4.1: A Deterministic Trend in Real GDP?



The Random Walk Model

$$y_t = y_{t-1} + \varepsilon_t \quad (\text{or } \Delta y_t = \varepsilon_t).$$

$$\text{Hence } y_t = y_0 + \sum_{i=1}^t \varepsilon_i$$

Given the first t realizations of the $\{\varepsilon_t\}$ process, the conditional mean of y_{t+1} is

$$E_t y_{t+1} = E_t(y_t + \varepsilon_{t+1}) = y_t$$

Similarly, the conditional mean of y_{t+s} (for any $s > 0$) can be obtained from

$$E_t y_{t+s} = y_t + E_t \sum_{i=1}^s \varepsilon_{t+i} = y_t$$

$$\text{var}(y_t) = \text{var}(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1) = t\sigma^2$$

$$\text{var}(y_{t-s}) = \text{var}(\varepsilon_{t-s} + \varepsilon_{t-s-1} + \dots + \varepsilon_1) = (t-s)\sigma^2$$



Random Walk Plus Drift

$$y_t = y_{t-1} + a_0 + \varepsilon_t$$

Given the initial condition y_0 , the general solution for y_t is

$$y_t = y_0 + a_0 t + \sum_{i=1}^t \varepsilon_i$$

$$y_{t+s} = y_0 + a_0(t+s) + \sum_{i=1}^{t+s} \varepsilon_i$$

$$E_t y_{t+s} = y_t + a_0 s.$$



The autocorrelation coefficient

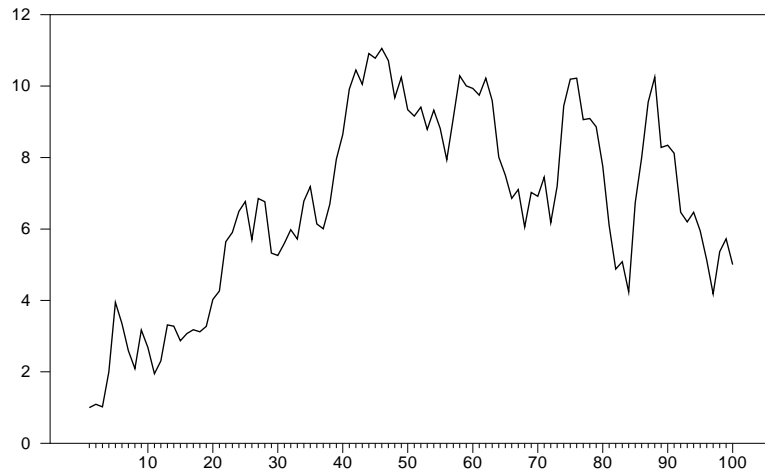
$$\begin{aligned} E[(y_t - y_0)(y_{t-s} - y_0)] &= E[(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1)(\varepsilon_{t-s} + \varepsilon_{t-s-1} + \dots + \varepsilon_1)] \\ &= E[(\varepsilon_{t-s})^2 + (\varepsilon_{t-s-1})^2 + \dots + (\varepsilon_1)^2] \\ &= (t-s)\sigma^2 \end{aligned}$$

$$\begin{aligned} \rho_s &= (t-s) / \sqrt{(t-s)t} \\ &= [(t-s)/t]^{0.5} \end{aligned}$$

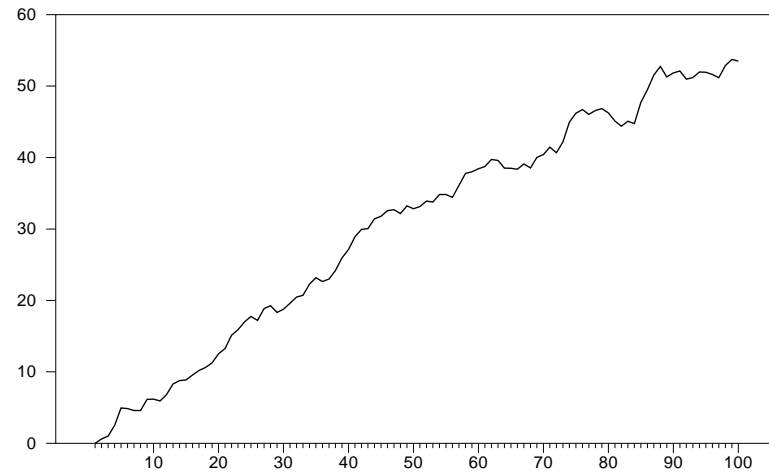
Hence, in using sample data, *the autocorrelation function for a random walk process will show a slight tendency to decay.*



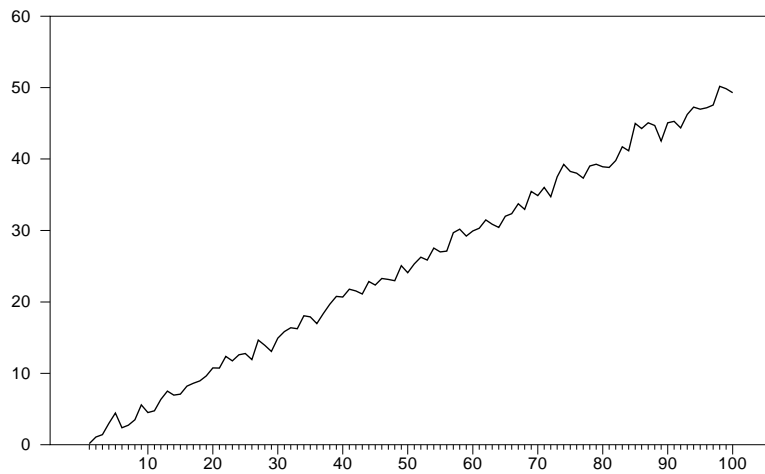
Panel (a): Random Walk



Panel (b): Random Walk Plus Drift



Panel (c): Trend Stationary



Panel (d): Random Walk Plus Noise

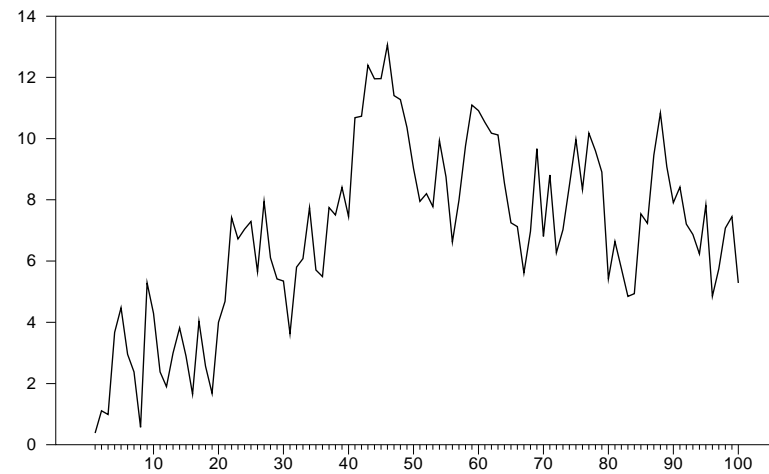


Figure 4.2: Four Series With Trends



Figure 4.3: The Business Cycle?

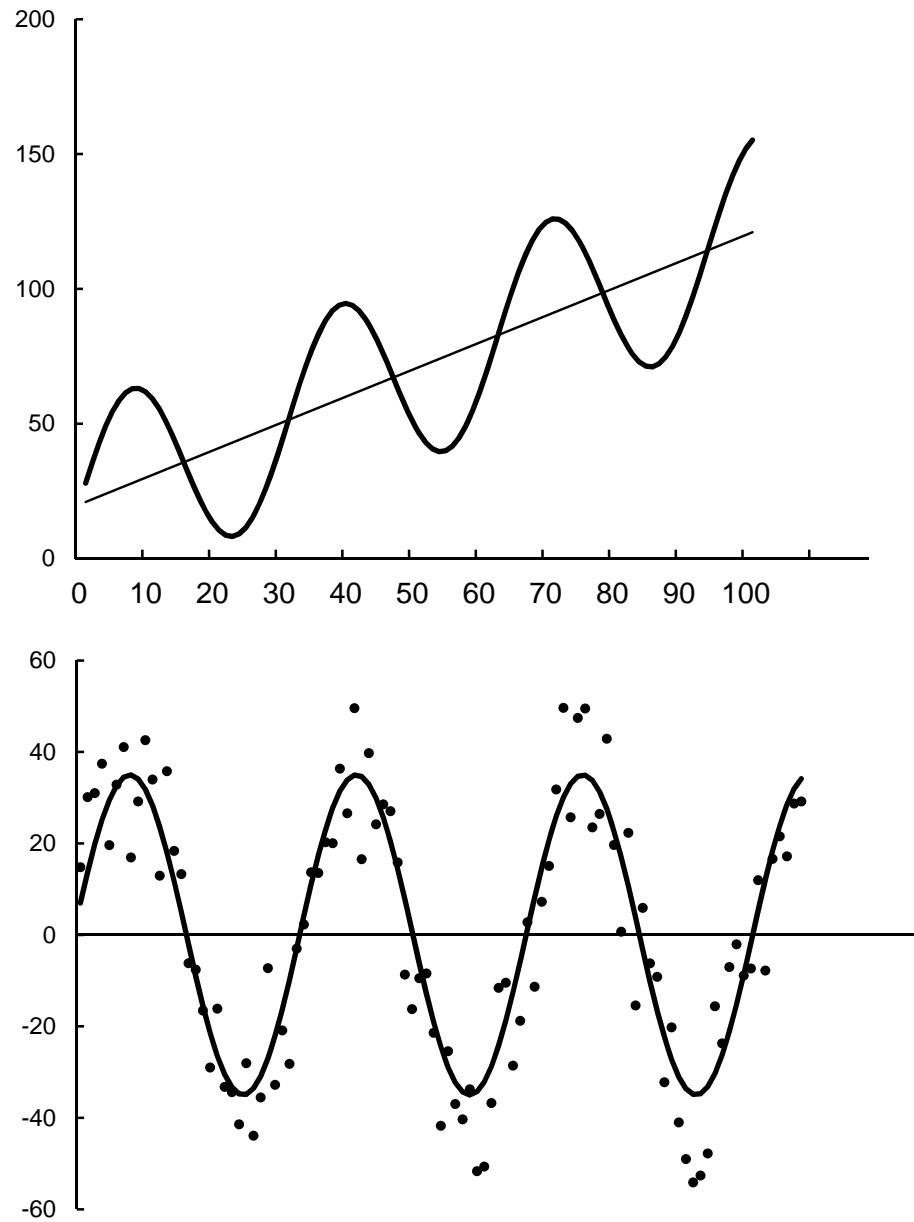


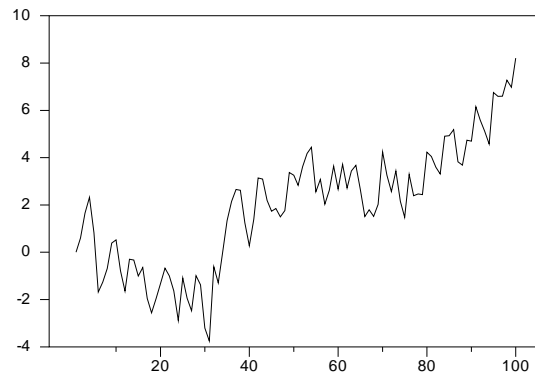
Table 4.1: Selected Autocorrelations From Nelson and Plosser

	ρ_1	ρ_2	$r(1)$	$r(2)$	$d(1)$	$d(2)$
Real GNP	.95	.90	.34	.04	.87	.66
Nominal GNP	.95	.89	.44	.08	.93	.79
Industrial Production	.97	.94	.03	-.11	.84	.67
Unemployment Rate	.75	.47	.09	-.29	.75	.46

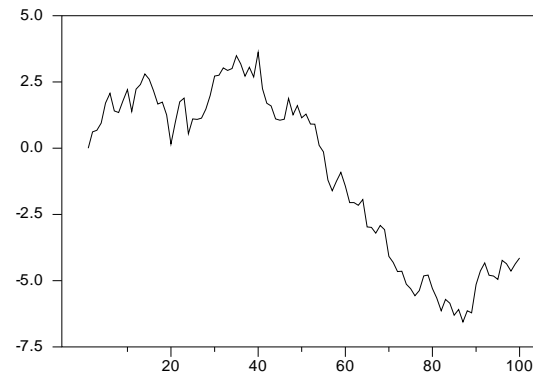
Worksheet 4.1

Consider the two random walk processes

$$y_t = y_{t-1} + \varepsilon_{yt}$$

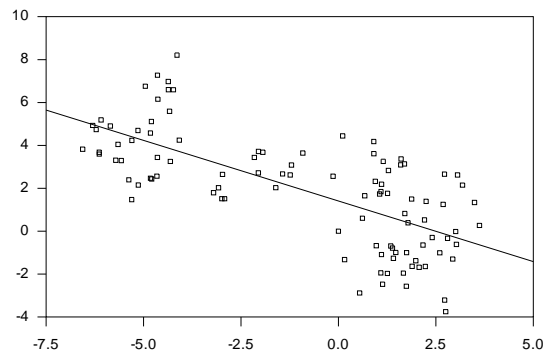


$$z_t = z_{t-1} + \varepsilon_{zt}$$

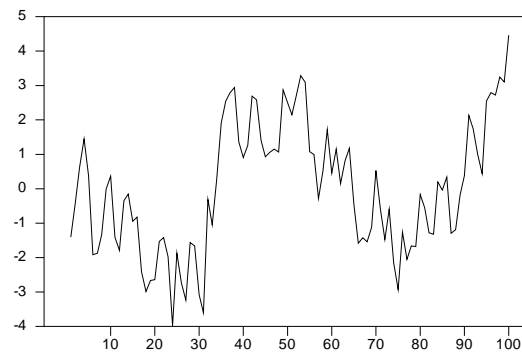


Since both series are unit-root processes with uncorrelated error terms, the regression of y_t on z_t is spurious. Given the realizations of $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$, it happens that y_t tends to increase as z_t tends to decrease. The regression line shown in the scatter plot of y_t on z_t captures this tendency. The correlation coefficient between y_t and z_t is -0.69 and a linear regression yields $y_t = 1.41 - 0.565z_t$. However, the residuals from the regression equation are nonstationary.

Scatter Plot of y_t Against z_t



Regression Residuals

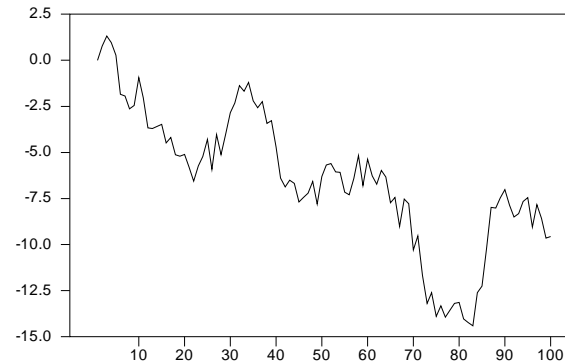
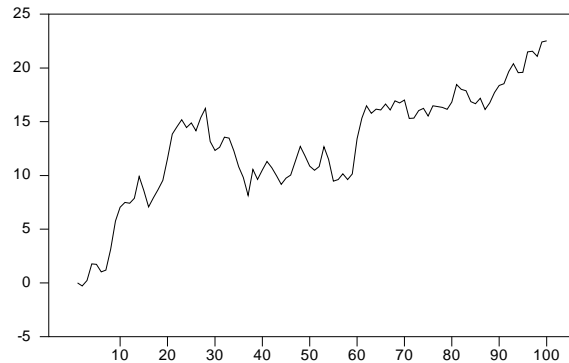


Worksheet 4.2

Consider the two random walk plus drift processes

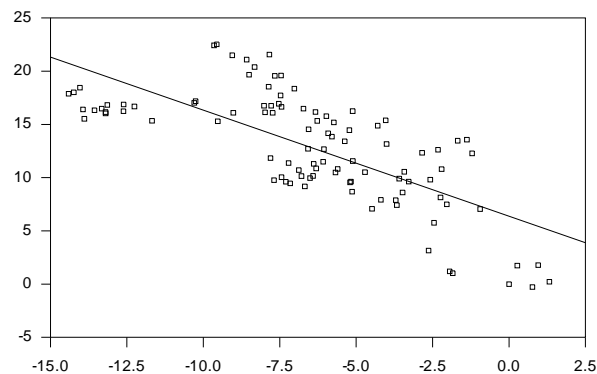
$$y_t = 0.2 + y_{t-1} + \varepsilon_{yt}$$

$$z_t = -0.1 + z_{t-1} + \varepsilon_{zt}$$

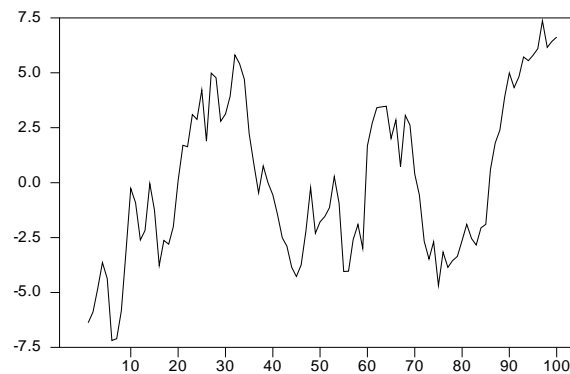


Here $\{y_t\}$ and $\{z_t\}$ series are unit-root processes with uncorrelated error terms so that the regression is spurious. Although it is the deterministic drift terms that cause the sustained increase in y_t and the overall decline in z_t , it appears that the two series are inversely related to each other. The residuals from the regression $y_t = 6.38 - 0.10z_t$ are nonstationary.

Scatter Plot of y_t Against z_t



Regression Residuals



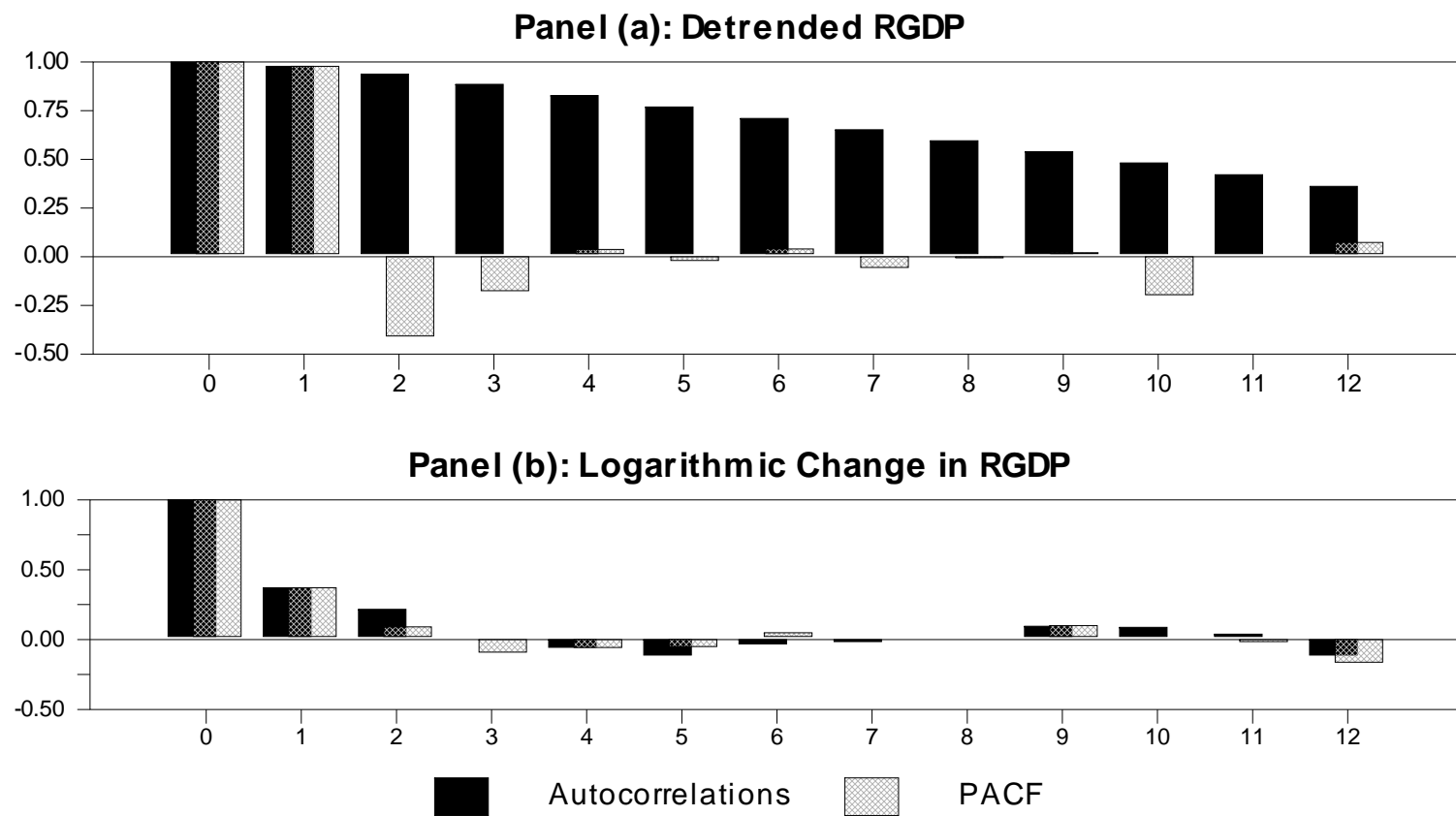


Figure 4.4 ACF and PACF

3. UNIT ROOTS AND REGRESSION RESIDUALS

- $y_t = a_0 + a_1 z_t + e_t$
- Assumptions of the classical model:
 - both the $\{y_t\}$ and $\{z_t\}$ sequences be stationary
 - the errors have a zero mean and a finite variance.
 - In the presence of nonstationary variables, there might be what Granger and Newbold (1974) call a **spurious regression**
 - A spurious regression has a high R^2 and t -statistics that appear to be significant, but the results are without any economic meaning.
 - The regression output “looks good” because the least-squares estimates are not consistent and the customary tests of statistical inference do not hold.



Four cases

- **CASE 1:** Both $\{y_t\}$ and $\{z_t\}$ are stationary.
 - the classical regression model is appropriate.
- **CASE 2:** The $\{y_t\}$ and $\{z_t\}$ sequences are integrated of different orders.
 - Regression equations using such variables are meaningless
- **CASE 3:** The nonstationary $\{y_t\}$ and $\{z_t\}$ sequences are integrated of the same order and the residual sequence contains a stochastic trend.
 - This is the case in which the regression is spurious.
 - In this case, it is often recommended that the regression equation be estimated in first differences.
- **CASE 4:** The nonstationary $\{y_t\}$ and $\{z_t\}$ sequences are integrated of the same order and the residual sequence is stationary.
 - In this circumstance, $\{y_t\}$ and $\{z_t\}$ are **cointegrated**.



The Dickey-Fuller tests

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

The ϕ_1 , ϕ_2 , and ϕ_3 statistics are constructed in exactly the same way as ordinary F -tests:

$$\phi_i = \frac{[SSR(restricted) - SSR(unrestricted)] / r}{SSR(unrestricted) / (T - k)}$$

Figure 4.6: The Dickey-Fuller Distribution

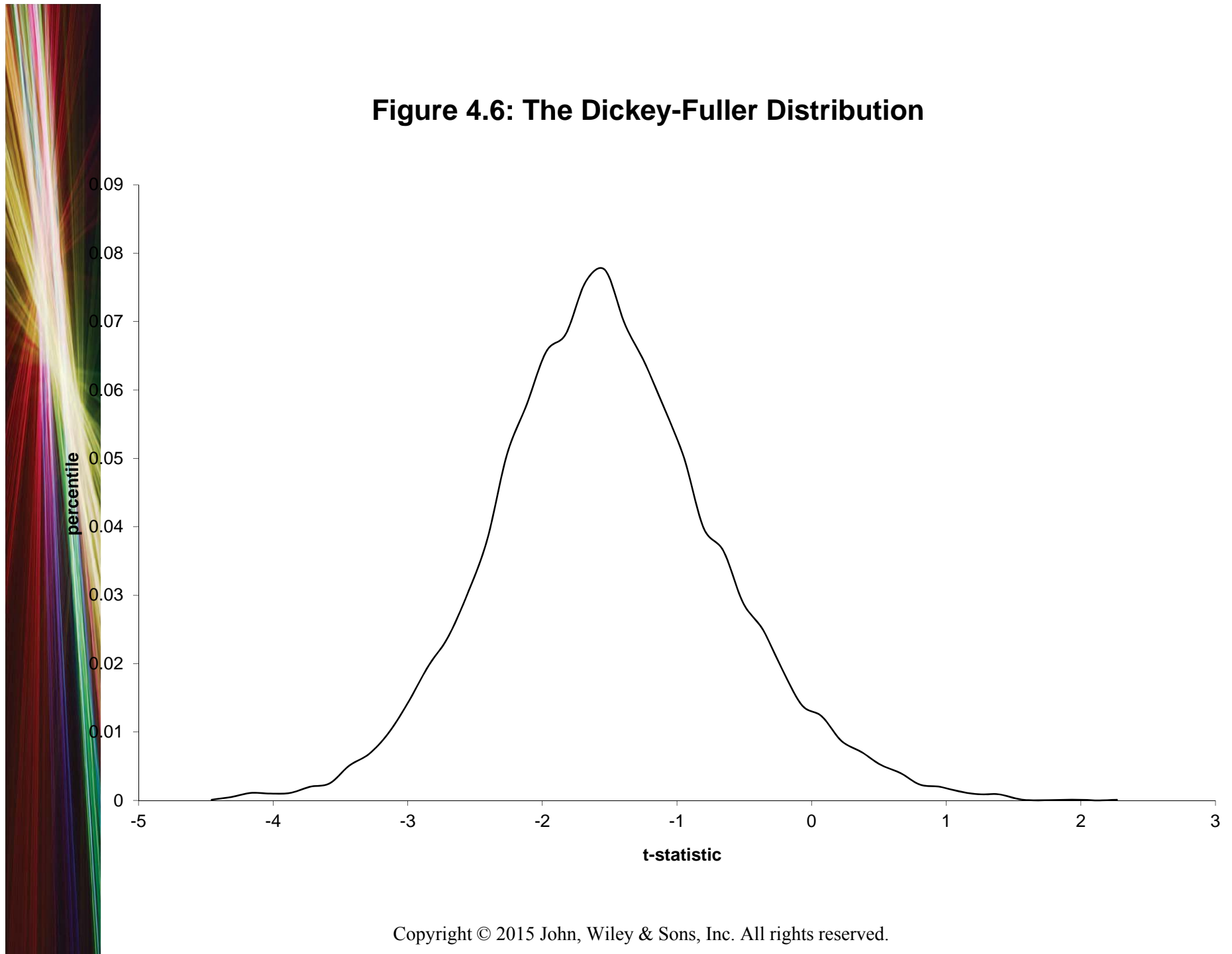


Table 4.2: Summary of the Dickey-Fuller Tests

Model	Hypothesis	Test Statistic	Critical values for 95% and 99% Confidence Intervals
$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \varepsilon_t$	$\gamma = 0$	τ_τ	-3.45 and -4.04
	$\gamma = a_2 = 0$	ϕ_3	6.49 and 8.73
	$a_0 = \gamma = a_2 = 0$	ϕ_2	4.88 and 6.50
$\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t$	$\gamma = 0$	τ_μ	-2.89 and -3.51
	$a_0 = \gamma = 0$	ϕ_1	4.71 and 6.70
$\Delta y_t = \gamma y_{t-1} + \varepsilon_t$	$\gamma = 0$	τ	-1.95 and -2.60

Table 4.3: Nelson and Plosser's Tests For Unit Roots

	p	a_0	a_2	γ	$\gamma + 1$
Real GNP	2	0.819 (3.03)	0.006 (3.03)	-0.175 (-2.99)	0.825
Nominal GNP	2	1.06 (2.37)	0.006 (2.34)	-0.101 (-2.32)	0.899
Industrial Production	6	0.103 (4.32)	0.007 (2.44)	-0.165 (-2.53)	0.835
Unemployment Rate	4	0.513 (2.81)	-0.000 (-0.23)	-0.294* (-3.55)	0.706

p is the chosen lag length. Entries in parentheses represent the t -test for the null hypothesis that a coefficient is equal to zero. Under the null of nonstationarity, it is necessary to use the Dickey-Fuller critical values. At the .05 significance level, the critical value for the t -statistic is -3.45.



Quarterly Real U.S. GDP

$$\Delta lrgdp_t = 0.1248 + 0.0001t - 0.0156lrgdp_{t-1} + 0.3663\Delta lrgdp_{t-1}$$

(1.58) (1.31) (-1.49) (6.26)

The t -statistic on the coefficient for $lrgdp_{t-1}$ is -1.49 . Table A indicates that, with 244 usable observations, the 10% and 5% critical value of τ_τ are about -3.13 and -3.43 , respectively. As such, we cannot reject the null hypothesis of a unit root.

The sample value of ϕ_3 for the null hypothesis $a_2 = \gamma = 0$ is 2.97. As Table B indicates that the 10% critical value is 5.39, we cannot reject the joint hypothesis of a unit root and no deterministic time trend. The sample value of ϕ_2 is 20.20. Since the sample value of ϕ_2 (equal to 17.61) far exceeds the 5% critical value of 4.75, we do not want to exclude the drift term. We can conclude that the growth rate of the real GDP series acts as a random walk plus drift plus the irregular term $0.3663\Delta lrgdp_{t-1}$.

Table 4.4: Real Exchange Rate Estimation

	γ	$H_0: \gamma=0$	Lags	Mean	ρ / DW	F	SD/ SEE
1973-1986							
Canada	-0.022 (0.016)	$t = -1.42$	0	1.05	0.059 1.88	0.194	5.47 1.16
Japan	-0.047 (0.074)	$t = -0.64$	2	1.01	-0.007 2.01	0.226	10.44 2.81
Germany	-0.027 (0.076)	$t = -0.28$	2	1.11	-0.014 2.04	0.858	20.68 3.71
1960-1971							
Canada	-0.031 (0.019)	$t = -1.59$	0	1.02	-0.107 2.21	0.434	.014 .004
Japan	-0.030 (0.028)	$t = -1.04$	0	0.98	0.046 1.98	0.330	.017 .005
Germany	-0.016 (0.012)	$t = -1.23$	0	1.01	0.038 1.93	0.097	.026 .004

EXTENSIONS OF THE DICKEY–FULLER TEST

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \dots + a_{p-2} y_{t-p+2} + a_{p-1} y_{t-p+1} + a_p y_{t-p} + \varepsilon_t$$

add and subtract $a_p y_{t-p+1}$ to obtain

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_{p-2} y_{t-p+2} + (a_{p-1} + a_p) y_{t-p+1} - a_p \Delta y_{t-p+1} + \varepsilon_t$$

Next, add and subtract $(a_{p-1} + a_p) y_{t-p+2}$ to obtain:

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \dots - (a_{p-1} + a_p) \Delta y_{t-p+2} - a_p \Delta y_{t-p+1} + \varepsilon_t$$

Continuing in this fashion, we obtain

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t$$

$$\gamma = - \left(1 - \sum_{i=1}^p a_i \right) \text{ and } \beta_i = - \sum_{j=i}^p a_j$$



Rule 1:

- Consider a regression equation containing a mixture of $I(1)$ and $I(0)$ variables such that the residuals are white noise. If the model is such that the coefficient of interest can be written as a coefficient on zero-mean stationary variables, then asymptotically, the OLS estimator converges to a normal distribution. As such, a t -test is appropriate.

- 
- Rule 1 indicates that you can conduct lag length tests using t-tests and/or F-tests on

$$\Delta y_t = \alpha_0 + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \dots + \beta_p \Delta y_{t-p+1} + \varepsilon_t$$



Selection of the Lag Length

- **general-to-specific** methodology
 - Start using a lag length of p^* . If the t -statistic on lag p^* is insignificant at some specified critical value, re-estimate the regression using a lag length of p^*-1 . Repeat the process until the last lag is significantly different from zero.
 - Once a tentative lag length has been determined, diagnostic checking should be conducted.
- Model Selection Criteria (AIC ,SBC)
- Residual-based LM tests



The Test with MA Components

- $A(L)y_t = C(L)\varepsilon_t$ so that $A(L)/C(L)y_t = \varepsilon_t$
- *So that* $D(L)y_t = \varepsilon_t$
 - Even though $D(L)$ will generally be an infinite-order polynomial we can use the same technique as used above to form the infinite-order autoregressive model
 - However, unit root tests generally work poorly if the error process has a strongly negative MA component.



Example of a Negative MA term

$$y_t = y_{t-1} + \varepsilon_t - \beta_1 \varepsilon_{t-1}; \quad 0 < \beta_1 < 1.$$

The ACF is:

$$\begin{aligned} \gamma_0 &= E[(y_t - y_0)^2] = \sigma^2 + (1 - \beta_1)^2 E[(\varepsilon_{t-1})^2 + (\varepsilon_{t-2})^2 + \dots + (\varepsilon_1)^2] \\ &= [1 + (1 - \beta_1)^2(t - 1)]\sigma^2 \end{aligned}$$

$$\begin{aligned} \gamma_s &= E[(y_t - y_0)(y_{t-s} - y_0)] \\ &= E[(\varepsilon_t + (1 - \beta_1)\varepsilon_{t-1} + \dots + (1 - \beta_1)\varepsilon_1)(\varepsilon_{t-s} + (1 - \beta_1)\varepsilon_{t-s-1} + \dots + (1 - \beta_1)\varepsilon_1)] \\ &= (1 - \beta_1) [1 + (1 - \beta_1)(t - s - 1)] \sigma^2 \end{aligned}$$

The ρ_i approach unity as the sample size t becomes infinitely large.

For the sample sizes usually found in applied work, the autocorrelations can be small.

Let β_1 be close to unity so that terms containing $(1 - \beta_1)^2$ can be safely ignored. The ACF can be approximated by $\rho_1 = \rho_2 = \dots = (1 - \beta_1)^{0.5}$. For example, if $\beta_1 = 0.95$, all of the autocorrelations should be 0.22.

Multiple Roots

- Consider

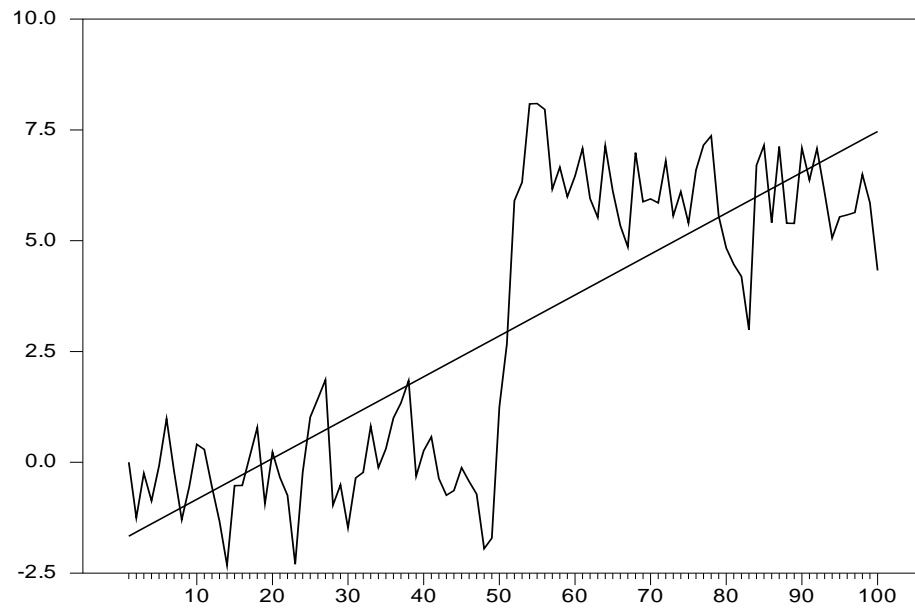
$$\Delta^2 y_t = a_0 + \beta_1 \Delta y_{t-1} + \varepsilon_t$$

If β_1 does differ from zero, estimate

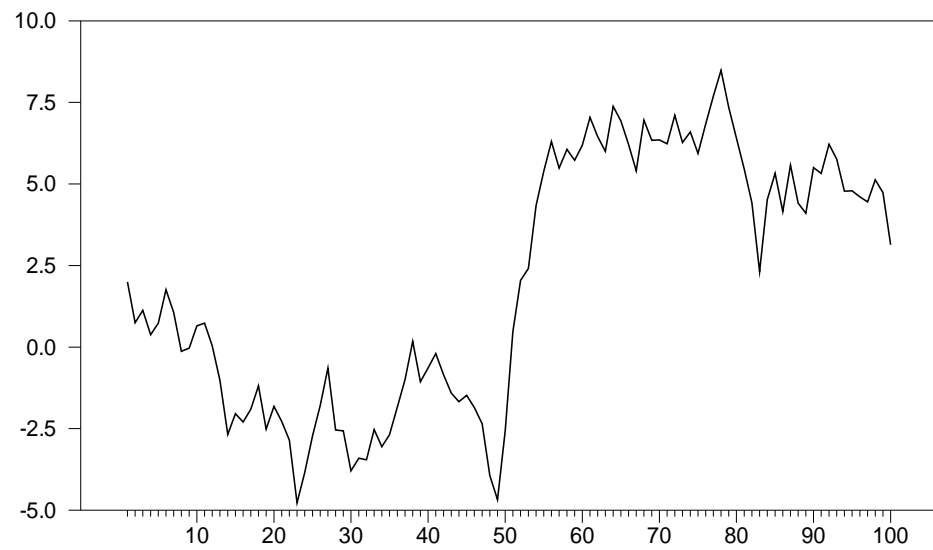
$$\Delta^2 y_t = a_0 + \beta_1 \Delta y_{t-1} + \beta_2 y_{t-1} + \varepsilon_t$$

If you reject the null hypothesis, $\beta_2 = 0$, conclude that $\{y_t\}$ is stationary.

Panel (a) $y_t = 0.5y_{t-1} + \varepsilon_t + D_L$



Panel (b) $y_t = y_{t-1} + \varepsilon_t + D_P$





Perron's Test

- Let the null be $y_t = a_0 + y_{t-1} + \mu_1 D_P + \mu_2 D_L + \varepsilon_t$
 - where D_P and D_L are the pulse and level dummies
- Estimate the regression (the alternative):
$$y_t = a_0 + a_2 t + m_1 D_P + m_2 D_L + m_3 D_T + \varepsilon_t$$
 - Let D_T be a trend shift dummy such that $D_T = t - \tau$ for $t > \tau$ and zero otherwise.
- Now consider a regression of the residuals

$$\hat{y}_t = a_1 \hat{y}_{t-1} + \varepsilon_{1t}$$

If the errors do not appear to be white noise, estimate the equation in the form of an augmented Dickey–Fuller test.

The t -statistic for the null hypothesis $a_1 = 1$ can be compared to the critical values calculated by Perron (1989). For $\lambda = 0.5$, Perron reports the critical value of the t -statistic at the 5 percent significance level to be -3.96 for H_2 and -4.24 for H_3 .

Table 4.6: Retesting Nelson and Plosser's Data For Structural Change

	T	λ	k	a_0	μ_1	μ_2	a_2	a_1
Real GNP	62	0.33	8	3.44 (5.07)	-0.189 (-4.28)	-0.018 (-0.30)	0.027 (5.05)	0.282 (-5.03)
Nominal GNP	62	0.33	8	5.69 (5.44)	-3.60 (-4.77)	0.100 (1.09)	0.036 (5.44)	0.471 (-5.42)
Industrial Prod.	111	0.66	8	0.120 (4.37)	-0.298 (-4.56)	-0.095 (-0.95)	0.032 (5.42)	0.322 (-5.47)

The appropriate t -statistics are in parenthesis. For a_0 , μ_1 , μ_2 , and a_2 , the null is that the coefficient is equal to zero. For a_1 , the null hypothesis is $a_1 = 1$. Note that all estimated values of a_1 are significantly different from unity at the 1% level.



Power

- Formally, the **power** of a test is equal to the probability of rejecting a false null hypothesis (i.e., one minus the probability of a type II error). The power for tau-mu is

α_1	10%	5%	1%
0.80	95.9	87.4	51.4
0.90	52.1	33.1	9.0
0.95	23.4	12.7	2.6
0.99	10.5	5.8	1.3



Nonlinear Unit Root Tests

- Enders-Granger Test

$$\Delta y_t = I_t \rho_1(y_{t-1} - \tau) + (1 - I_t) \rho_2(y_{t-1} - \tau) + \varepsilon_t$$

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} \geq \tau \\ 0 & \text{if } y_{t-1} < \tau \end{cases}$$

- LSTAR and ESTAR Tests
- Nonlinear Breaks—Endogenous Breaks



Schmidt and Phillips (1992) LM Test

- The overly-wide confidence intervals for γ means that you are less likely to reject the null hypothesis of a unit root even when the true value of γ is not zero. A number of authors have devised clever methods to improve the estimates of the intercept and trend coefficients.

$$y_t = a_0 + a_2 t + \sum_{i=1}^t \varepsilon_t$$

$$\Delta y_t = a_2 + \varepsilon_t$$

- The idea is to estimate the trend coefficient, a_2 , using the regression $\Delta y_t = a_2 + \varepsilon_t$. As such, the presence of the stochastic trend $\sum \varepsilon_i$ does not interfere with the estimation of a_2 .



LM Test Continued

- Use this estimate to form the detrended series as

$$y_t^d = y_t - (y_1 - \hat{a}_2) - \hat{a}_2 t$$

- Then use the detrended series to estimate

$$\Delta y_t = a_0 + \gamma y_{t-1}^d + \sum_{i=1}^p c_i \Delta y_{t-i}^d + \varepsilon_t$$

- Schmidt and Phillips (1992) show that it is preferable to estimate the parameters of the trend using a model without the persistent variable y_{t-1} .
- Elliott, Rothenberg and Stock (1996) show that it is possible to further enhance the power of the test by estimating the model using something close to first-differences.

The Elliott, Rothenberg, and Stock Test

Instead of creating the first difference of y_t , Elliott, Rothenberg and Stock (ERS) preselect a constant close to unity, say α , and subtract αy_{t-1} from y_t to obtain:

$$\begin{aligned}\tilde{y}_t &= a_0 + a_2 t - \alpha a_0 - \alpha a_2 (t-1) + e_t, \quad \text{for } t = 2, \dots, \\ &= (1 - \alpha)a_0 + a_2[(1 - \alpha)t + \alpha] + e_t \\ &= a_0 z1_t + a_2 z2_t + e_t\end{aligned}$$

$$z1_t = (1 - \alpha) ; z2_t = \alpha + (1 - \alpha)t.$$

The important point is that the estimates a_0 and a_2 can be used to detrend the $\{y_t\}$ series

$$\Delta y_t^d = \gamma y_{t-1}^d + \sum_{i=1}^p c_i \Delta y_{t-i}^d + \varepsilon_t$$



Panel Unit Root Tests

- $$\Delta y_{it} = a_{i0} + \gamma_i y_{it-1} + a_{i2} \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + \varepsilon_{it}$$
- One way to obtain a more powerful test is to pool the estimates from a number separate series and then test the pooled value. The theory underlying the test is very simple: if you have n independent and unbiased estimates of a parameter, the mean of the estimates is also unbiased. More importantly, so long as the estimates are independent, the central limit theory suggests that the sample mean will be normally distributed around the true mean.
 - The difficult issue is to correct for cross equation correlation
- Because the lag lengths can differ across equations, you should perform separate lag length tests for each equation. Moreover, you may choose to exclude the deterministic time trend. However, if the trend is included in one equation, it should be included in all

Table 4.8: The Panel Unit Root Tests for Real Exchange Rates

	Lags	Estimated γ_i	t-statistic		Estimated γ_i	t-statistic
	Log of the Real Rate				Minus the Common Time Effect	
Australia	5	-0.049	-1.678		-0.043	-1.434
Canada	7	-0.036	-1.896		-0.035	-1.820
France	1	-0.079	-2.999		-0.102	-3.433
Germany	1	-0.068	-2.669		-0.067	-2.669
Japan	3	-0.054	-2.277		-0.048	-2.137
Netherlands	1	-0.110	-3.473		-0.137	-3.953
U.K.	1	-0.081	-2.759		-0.069	-2.504
U.S.	1	-0.037	-1.764		-0.045	-2.008



Limitations

- The null hypothesis for the IPS test is $\gamma_i = \gamma_2 = \dots = \gamma_n = 0$. Rejection of the null hypothesis means that *at least* one of the γ_i 's differs from zero.
- At this point, there is substantial disagreement about the asymptotic theory underlying the test. Sample size can approach infinity by increasing n for a given T , increasing T for a given n , or by simultaneously increasing n and T .
 - For small T and large n , the critical values are dependent on the magnitudes of the various β_{ij} .
- The test requires that the error terms be serially uncorrelated and contemporaneously uncorrelated.
 - You can determine the values of p_i to ensure that the autocorrelations of $\{\varepsilon_{it}\}$ are zero. Nevertheless, the errors may be contemporaneously correlated in that $E\varepsilon_{it}\varepsilon_{jt} \neq 0$
 - The example above illustrates a common technique to correct for correlation across equations. As in the example, you can subtract a common time effect from each observation. However, there is no assurance that this correction will completely eliminate the correlation. Moreover, it is quite possible that is nonstationary. Subtracting a nonstationary component from each sequence is clearly at odds with the notion that the variables are stationary.



The Beveridge-Nelson Decomposition

- The trend is defined to be the conditional expectation of the limiting value of the forecast function. In lay terms, the trend is the “long-term” forecast. This forecast will differ at each period t as additional realizations of $\{e_t\}$ become available. At any period t , the stationary component of the series is the difference between y_t and the trend μ_t .



BN 2

- Estimate the $\{y_t\}$ series using the Box–Jenkins technique.
 - After differencing the data, an appropriately identified and estimated ARMA model will yield high-quality estimates *of the coefficients*.
- Obtain the one-step-ahead forecast errors of $E_t y_{t+s}$ for large s . Repeating for each value of t yields the entire set of premanent components
- The irregular component is y_t minus the value of the trend.



The HP Filter

Let the trend of a nonstationary series be the $\{\mu_t\}$ sequence so that $y_t - \mu_t$ the stationary component

$$\frac{1}{T} \sum_{t=1}^T (y_t - \mu_t)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} [(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})]^2$$

For a given value of λ , the goal is to select the $\{\mu_t\}$ sequence so as to minimize this sum of squares. In the minimization problem λ is an arbitrary constant reflecting the “cost” or penalty of incorporating fluctuations into the trend.

In applications with quarterly data, including Hodrick and Prescott (1984) λ is usually set equal to 1,600.

Large values of λ acts to “smooth out” the trend.

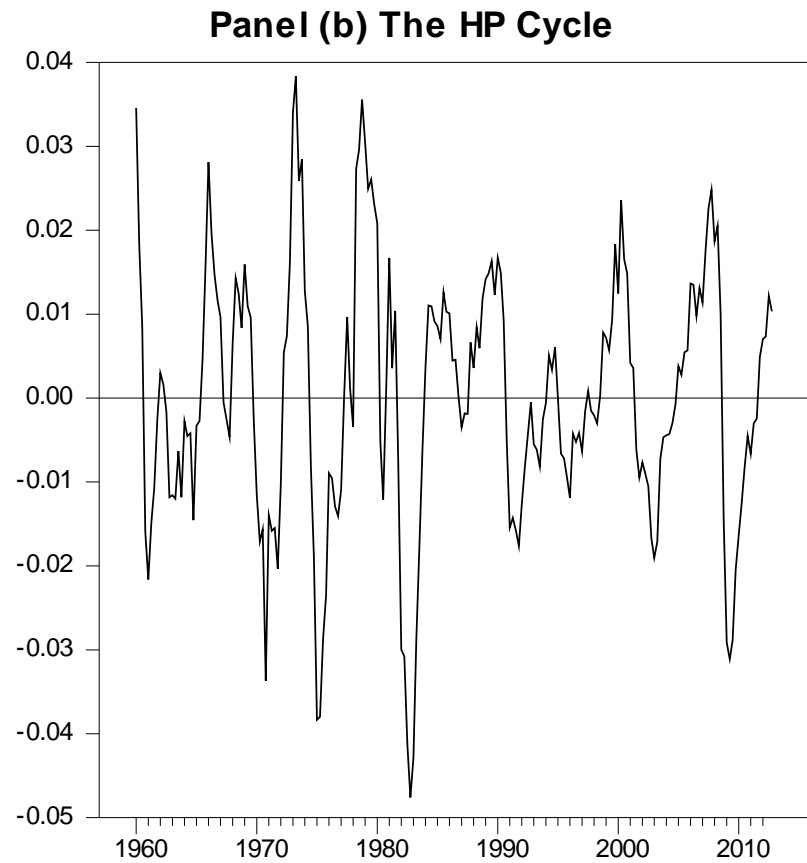
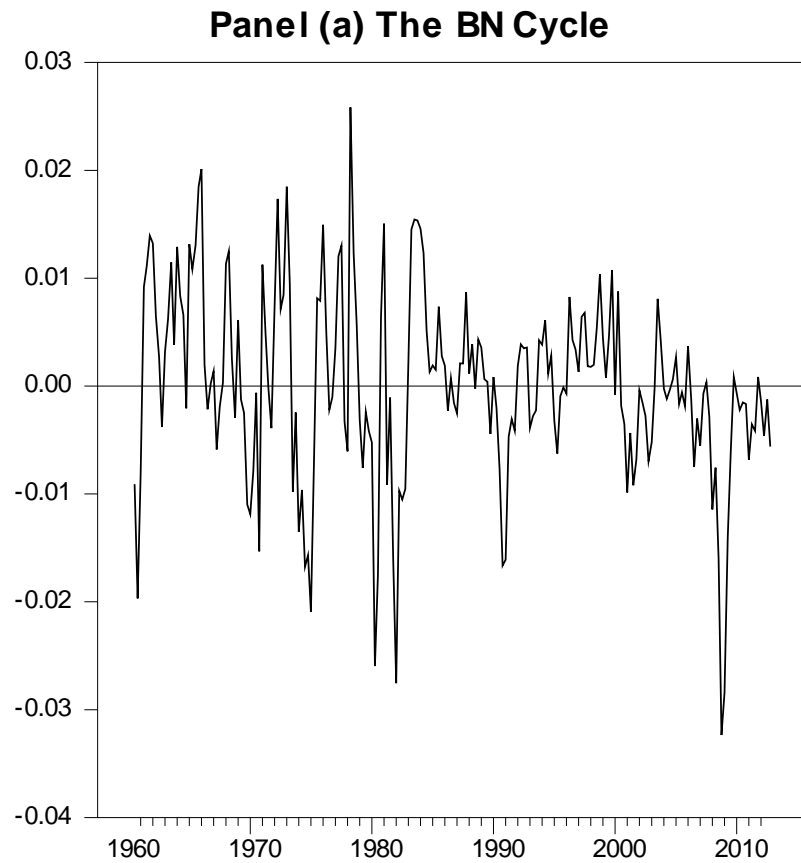


Figure 4.11: Two Decompositions of GDP

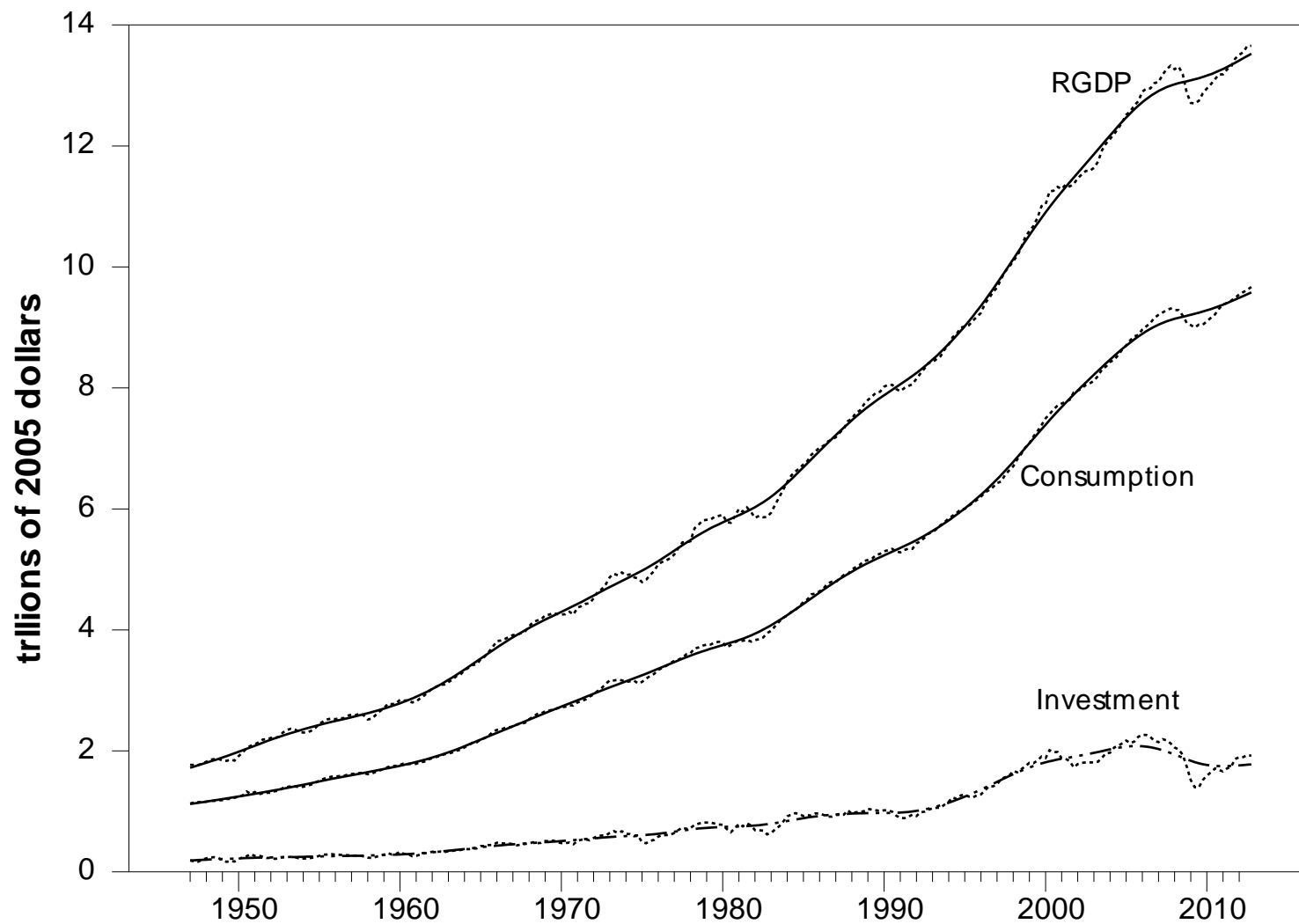


Figure 4.12: Real GDP, Consumption and Investment